The Cox-Ross-Rubinstein (CRR) model, introduced in their 1979 *Journal of Financial Economics* paper, is a binomial option pricing model based on risk neutral valuation. The formulas for pricing European call and put options in the CRR model with n time steps are given by:

$$C = e^{-rT} \sum_{j=a}^{n} \frac{n!}{j!(n-j)!} \, q^j (1-q)^{n-j} C_j, \text{ and}$$

$$P = e^{-rT} \sum_{j=0}^{a-1} \frac{n!}{j!(n-j)!} \, q^j (1-q)^{n-j} P_j, \text{ where}$$

- C is the arbitrage-free price of the call option.

- P is the arbitrage-free price of the put option.

- S is the underlying asset's current price.

- K is the option's exercise price.

- σ is the underlying asset's volatility.

- δt is the length of each time step.

- n is the number of time steps.

- $T = n\delta t =$ time to expiration.

- $u = e^{\sigma\sqrt{\delta t}}$ is the factor by which the asset price increases in an upward movement.

- $d = 1/u$ is the factor by which the asset price decreases in a downward movement.

- r is the annualized riskless rate of interest.

- $q = \dfrac{e^{r\delta t} - d}{u - d}$ is the probability of an up movement in the underlying asset price.

- $1 - q$ is the probability of a down movement in the underlying asset price.

- a corresponds to the minimum number of up moves required in order for the call option to end up in the money and is found by rounding to the nearest integer greater than $b = \ln\left(K/Sd^n\right)/\ln\left(u/d\right)$.[1]

The CRR model consists of four key components:

- $e^{-rT} =$ the present value of \$1;

- $q^j(1-q)^{n-j} =$ the risk neutral probability of one path sequence ending up at the $j, n-j$ terminal mode;

- $\dfrac{n!}{j!\,(n-j)!}$ indicates the total number of unique path sequences ending up at the $j, n-j$ terminal mode;

- $C_j = u^j d^{n-j} S - K$ and $P_j = K - u^j d^{n-j} S$ corresponds to the in-the-money call and put option payoffs at the $j, n-j$ terminal mode.

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[1]The call option is in the money at all $n-a+1$ terminal nodes where there have been a or more up moves, whereas the put option is in the money at all terminal nodes where there have been $a-1$ or less up moves.