Synopsis of Hull's Properties of Stock Options chapter

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Hull's "Properties of Stock Options" chapter mostly enumerates various "theorems" concerning boundaries on American and European call and put option price. These "theorems" appear (in the textbook and in my lecture on this topic) in the following order:

1. A call option is never worth more than the underlying stock.

2. A put option is never worth more than the exercise price.

3. A European put option is never worth more than the present value of the strike price.

4. Options never have negative value.

5. American options are at least as valuable as European options.

6. American options with more time to maturity are at least as valuable as the same options with less time to maturity.

7. European call options with more time to maturity are at least as valuable as the same options with less time to maturity (no such property exists for European puts).

8. An American option is worth at least its exercised value (the payoff you receive if you exercise today; no such restriction exists for European options because exercise may only occur at date T).

9. Since $C(S, K, t, T) \geq c(S, K, t, T)$ (see theorem 5), the price of a (American or European) call option must be greater than or equal to $max[0, S(t) - Ke^{-r(T-t)}]$; this theorem is proven by invoking the principle of arbitrage-free pricing; i.e., if the $max[0, S(t) - Ke^{-r(T-t)}]$ boundary were breached, then this would give rise to riskless arbitrage opportunities.

10. Since $P(S, K, t, T) \geq p(S, K, t, T)$(see theorem 5), the price of a (American or European) put option must be greater than or equal to $max[0, Ke^{-r(T-t)} - S(t)]$; this theorem is proven by invoking the principle of arbitrage-free pricing; i.e., if the $max[0, Ke^{-r(T-t)} - S(t)]$ boundary were breached, then this would give rise to riskless arbitrage opportunities.

Since theorems 1 and 9 jointly imply that $max[0, S(t) - Ke^{-r(T-t)}] \leq c(t) \leq S(t)$, this means that the value of a call option on a non-dividend paying stock must lie within the shaded region shown below; otherwise, riskless arbitrage would exist:[1]

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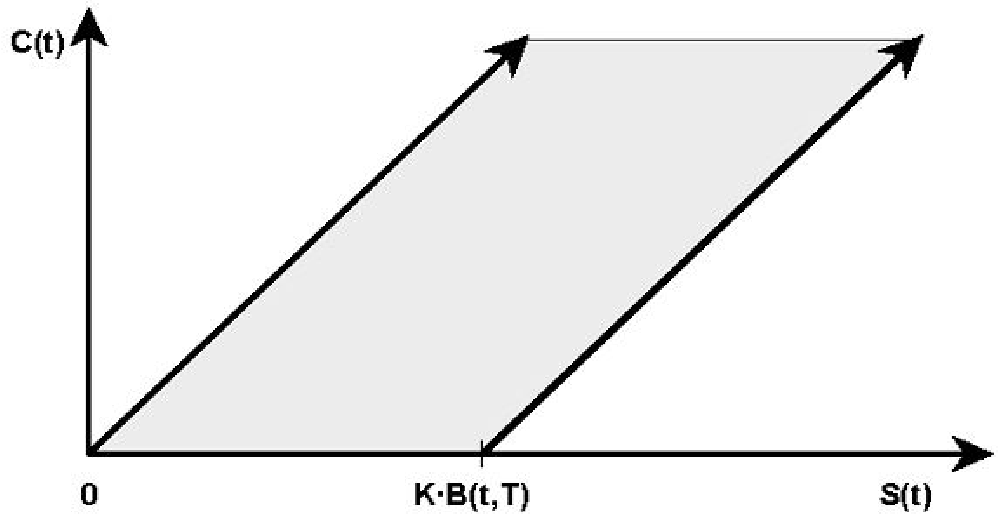
[1] The term $B(t, T)$ shown in the graphs on the following page corresponds to $Ke^{-r(T-t)}$.



Finally, since theorems 2 and 10 jointly imply that $max[0, Ke^{-r(T-t)} - S(t)] \leq p(t) \leq Ke^{-r(T-t)}$, this means that the value of a put option on a non-dividend paying stock must lie within the shaded region shown below; otherwise, riskless arbitrage would exist: