FINANCE 4366: SOLUTIONS FOR OPTIONS CLASS PROBLEMS, FEBRUARY 5, 2019

Problem #1

Suppose that you are interested in synthetically replicating a share of (a non-dividend paying) stock by implementing a trading strategy involving a call option, a put option, and a riskless discount bond. The exercise price for the two options is $50, and the riskless rate of interest is 5%. The options are both European and expire 1 year from today. The call and put options both currently trade for $4 each.

A. Describe a trading strategy involving the call, the put, and the riskless bond that will enable you to synthetically replicate a share of stock.

   SOLUTION: According to the put-call parity theorem, it is possible to synthetically replicate a share of stock by purchasing the call, selling the put, and buying a riskless bond worth $e^{-rT}K$. I.e., since

$$c + e^{-rT}K = p + S,$$

   it follows that

$$S = c + e^{-rT}K - p.$$

B. What is the value of your synthetic share of stock?

   SOLUTION: The value of the synthetic share of stock is $S = c + e^{-rT}K - p$ $= \$4 + \$47.56 - \$4 = \$47.56.$

C. Suppose the actual stock currently sells for $45. Describe an arbitrage strategy that will enable you to make riskless profits with zero net investment, and calculate the profit that you would earn from implementing such a strategy.

   SOLUTION: Since the synthetic share is more expensive than the real share, I can make riskless profits with zero net investment by simply short selling the synthetic share (i.e., selling the call, buying the put, and selling the bond) and using the proceeds from the short sale to purchase the real share. This is a perfectly hedged position which generates profit of $2.56 per share.

Problem #2

Suppose the riskless rate of interest is 0%, the price of a riskless bond is \$100, and the price of a (non-dividend paying) stock is \$100. In the future, only two equally probable outcomes exist for the economy, good and bad. In the good state, the stock is worth \$150, whereas in the bad state, the stock is worth \$75.

A. What is the "no arbitrage" price of a call option on the stock with an exercise price of \$100?

SOLUTION:

Note that the value of the call option is equal to the value of the replicating portfolio for the call option in both the future good and bad states; i.e.,

$$c_{1,g} = V_{1,g} = \max[0,150-100] = \$50 = \Delta 150 + \beta(100); \text{ and}$$
$$c_{1,b} = V_{1,b} = \max[0,75-100] = \$0 = \Delta 75 + \beta(100).$$

Since we have two equations in two unknowns, we can solve for Δ and β by subtracting the equation for $V_{1,b}$ from the equation for $V_{1,g}$:

$$\Delta 150 + \beta(100) - (\Delta 75 + \beta(100)) = 50;$$
$$\therefore \Delta 75 = 50 \Rightarrow \Delta = 2/3 \Rightarrow \beta = -.5.$$

If I hold a portfolio consisting of 2/3 of a share of stock and a short position in half of one bond, then in the good state, this portfolio pays off (2/3)(150) – (.5)(100) = \$50, and in the bad state, this portfolio pays off (2/3)(75) – (.5)(100) = \$0. This replicates the payoff on the call option; since $c(T) = \max(0, S(T) - K),$ in the good state the call pays off $c_{1,g} = \max[0,150-100] = \$50$ and in the bad state the call pays off $c_{1,b} = \max[0,50-100] = \$0.$ Since the replicating portfolio (which consists of 2/3 of a share of stock and a short position in half of one bond) is worth (2/3)(100)-(.5)(100) = \$16.67, then the call option *must* be worth this same amount; otherwise riskless arbitrage profits could be earned.

B. What is the "no arbitrage" price of a put option on the stock with an exercise price of \$100?

SOLUTION:

The simplest way to solve this problem is to simply note that since the "no arbitrage" price of a call option on the stock with an exercise price of \$100 is \$16.67, it follows from the put-call parity theorem that the "no arbitrage" price of a put option on the stock with an exercise price of \$100 is $p = c + e^{-rT}K - S$ = \$16.67 + \$100 - \$100 = \$16.67.

The more complicated (but equally correct) method for answering this question is to apply the same solution procedure as we did for pricing the call option. Note that the value of the put option is equal to the value of the replicating portfolio for the put option in both the future good and bad states; i.e.,

$$p_{1,g} = V_{1,g} = \max[0, 100 - 150] = \$0 = \Delta 150 + \beta(100); \text{ and}$$
$$p_{1,b} = V_{1,b} = \max[0, 100 - 75] = \$25 = \Delta 75 + \beta(100).$$

Since we have two equations in two unknowns, we can solve for Δ and β by subtracting the equation for $V_{1,b}$ from the equation for $V_{1,b}$:

$$\Delta 150 + \beta(100) - (\Delta 75 + \beta(100)) = -25;$$
$$\therefore \Delta 75 = -25 \Rightarrow \Delta = -1/3 \Rightarrow \beta = .5.$$

If I hold a portfolio consisting of a short position in 1/3 of a share of stock and a long position in half of one bond, then in the good state, this portfolio pays off (-1/3)(150) + (.5)(100) = \$0, and in the bad state, this portfolio pays off (-1/3)(75) + (.5)(100) = \$25. This replicates the payoff on the put option; since $p(T) = \max(0, K - S(T))$, in the good state the put pays off $p_{1,g} = \max[0, 100 - 150] = \$0$ and in the bad state the put pays off $p_{1,b} = \max[0, 100 - 75] = \$25$. Since the replicating portfolio (which consists of a short position in 1/3 of a share of stock and a long position in half of one bond) is worth (-1/3)(100)+(.5)(100) = \$16.67, then the put option *must* be worth this same amount; otherwise riskless arbitrage profits could be earned.